ANALYSIS OF SHOCK MOTION IN STBLI USING DNS DATA

Minwei Wu and Pino Martin
Department of Mechanical and Aerospace Engineering, Princeton University, Princeton, NJ 08544, USA

Abstract Direct numerical simulation data of a 24° compression ramp configuration are used to analyze the shock motion. The motion can be observed from wall-pressure and mass-flux signals measured in the free stream. The characteristic low frequency is in the range of 0.007-0.013 \( U_\infty / \delta \), as found in Wu & Martin (2007). The shock motion also exhibits high-frequency of \( O(U_\infty / \delta) \), small-amplitude spanwise wrinkling, which is mainly caused by the spanwise nonuniformity of turbulent structures in the incoming boundary layer. In studying the low frequency streamwise characteristic low frequency is in the range of 0.007-0.013 \( U_\infty / \delta \), as found in Wu & Martin (2007). The shock motion also exhibits high-frequency of \( O(U_\infty / \delta) \), small-amplitude spanwise wrinkling, which is mainly caused by the spanwise nonuniformity of turbulent structures in the incoming boundary layer. In studying the low frequency streamwise oscillation, conditional statistics show that there is no significant difference in the properties of the incoming boundary layer when the shock location is upstream or downstream. The spanwise-mean separation point also undergoes a low frequency motion and is found to highly correlate with the shock motion. A small correlation is found between the low-momentum structures in the incoming boundary layer and the separation point. Correlations among the spanwise-mean separation point, reattachment point and the shock location indicate that the low-frequency shock unsteadiness is influenced by the downstream flow.

INTRODUCTION

The boundary layer flow over a compression ramp is one of the canonical shock wave and turbulent boundary layer interaction (STBLI) configurations that have been studied extensively in experiments since the 1970’s. From this body of work, we have learned that the shock motion has a frequency much lower than the characteristic frequency of the incoming boundary layer. The time scale of the low frequency motion is \( O(10^6/U_\infty) \), as reported in various experiments such as Dolling & Or (1985), Selig (1988), Dussauge et al. (2006), and Dupont et al. (2006). In contrast, the characteristic time scale of the incoming boundary layer is \( O(\delta/U_\infty) \). The scale to normalize the frequency of the shock is still under debate. However, Dussauge et al. (2006) found that using \( St_L = fL/U_\infty \), where \( L \) is the streamwise length of the separation bubble, experimental data (covering a wide range of Mach numbers and Reynolds numbers and various configurations) can be grouped between \( St_L = 0.02 \) and 0.05. Also, the cause of the low frequency motion is still a research question. Plotkin (1975) proposed a damped spring model for the shock motion. Andreopoulos & Muck (1987) concluded that the shock motion is driven by the bursting events in the incoming boundary layer. However, Thomas et al. (1994) found no connection between the shock motion and bursting events in the incoming boundary layer. However, Thomas et al. (1994) found no connection between the shock motion and bursting events in the incoming boundary layer. Erengil & Dolling (1991) found that there was a correlation between certain shock motions with pressure fluctuations in the incoming boundary layer. Beresh et al. (2002) found that positive velocity fluctuations near the wall correlate with downstream shock motion. Pirozzoli & Grasso (2006) analyzed DNS data of a reflected shock interaction and proposed that a resonance mechanism might be responsible for the shock unsteadiness. Dussauge et al. (2006) suggested that the three-dimensional nature of the interaction in the reflected shock configuration is a key to understanding the shock unsteadiness. Ganapathisubramani et al. (2006) proposed that very long alternating structures of uniform low-speed and high-speed fluid in the logarithmic region of the incoming boundary layer are responsible for the low frequency motion of the shock. These so called ‘superstructures’ have been observed in supersonic boundary layers by Samimy et al. (1994) and are also evident in the elongated wall-pressure correlation measurements of Owen & Horstmann (1972). Superstructures have also been observed in the atmospheric boundary layer experiments of Hutchins & Marusic (2007) and confirmed in DNS of supersonic boundary layers by Ringnette et al. (2007).

Wu & Martin (2007) presented a direct numerical simulation of STBLI for a 24° compression ramp configuration at Mach 2.9 and Reynolds number based on momentum thickness of 2300. They validated the DNS data against the experiments of Bookey et al. (2005) at matching flow conditions, and they illustrated the existence of the superstructures. In this paper, we use the Wu & Martin (2007) data to analyze the shock unsteadiness. While in previous experiments the shock motion is usually inferred by measuring the wall pressure, our analyses of the shock motion are carried mainly in the outer part of the boundary layer and in the free stream. This is because the Reynolds number that we consider is much lower than those in typical experiments. In turn, viscous effects are more prominent, the shock does not penetrate as deeply as in higher Reynolds number flows, and the shock location is not well-defined in the lower half of the boundary layer. In addition, the motion of the separation bubble is studied. Table 1 lists the inflow boundary layer conditions, and Figure 1 shows the computational domain and the coordinate system. Notice that we use \( z_n \) to denote the wall-normal coordinate and prime symbols to denote fluctuating quantities. Statistics are gathered over 3000\( U_\infty \). The characterization of the shock motion and the unsteadiness of the separation bubble are presented. A discussion is also presented before the conclusion section.

<table>
<thead>
<tr>
<th>M</th>
<th>Re_\infty</th>
<th>( \theta ) (mm)</th>
<th>( \delta^+ ) (mm)</th>
<th>( \delta ) (mm)</th>
<th>( \delta^+ )</th>
<th>( C_f )</th>
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<td>1.80</td>
<td>6.4</td>
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Table 1: Inflow conditions for the DNS. The Mach number, Reynolds number based on the momentum thickness, displacement thickness, and boundary layer thickness, boundary layer thickness in wall variables, and skin friction are given in order of appearance.
SHOCK MOTION

Figure 2(a) plots three wall-pressure signals measured at three streamwise locations upstream of the ramp corner (the corner is located at $x = 0$) along the spanwise center line. In the incoming boundary layer at $x = -6.9\delta$, the normalized magnitude is around one with small fluctuations. At $x = -2.98\delta$, which is the mean separation point (defined as the point where the mean skin friction coefficient changes sign from positive to negative), the magnitude fluctuates between 1 to 1.2. At $x = -2.18\delta$, the magnitude oscillates between 1.5 and 2. The corresponding premultiplied energy spectra are plotted in Figure 2(b). At the mean separation point, the peak frequency is $0.007U_\infty/\delta$. At $x = -2.18$, the peak is in $0.01 U_\infty/\delta$. Let us define the Strouhal number $St = fL/U_\infty$, where $L$ is the separation length ($L = 4.2\delta$ in the DNS). The range of $St_L$ is 0.03-0.042, which is consistent with the range given by Dussauge et al. (2006).

Contours of the magnitude of pressure on streamwise-spanwise planes are plotted in Figure 3. Two instantaneous flow fields are plotted at $z_n = 0.9\delta$ and $2\delta$ away from the wall. At $z_n = 0.9$, Figures 3(a) and (b), the shock is nearly uniform in the spanwise direction. The streamwise movement of the shock is roughly $1\delta$. Figures 3(c) and (d) plot the same times at a plane closer to the wall. We observe a wrinkling of the shock in the spanwise direction, with an amplitude of about $0.5\delta$. At $z_n = 0.9\delta$, the shock also moves in the streamwise direction in the same manner as shown in Figures 3(a) and (b). The amplitude of the motion in the streamwise direction is twice that of the spanwise wrinkling.

We analyze the shock motion within the context of these two aspects. One is that the shock wrinkles along the spanwise direction. The other corresponds to the larger amplitude motion upstream and downstream. The motion that is inferred from the wall-pressure signal in Figure 2 results from the combination of these two aspects. However, the low frequency motion is related to the large-amplitude, streamwise motion rather than to the spanwise wrinkling. This can be seen from the mass-flux signals measured in the free stream as shown in Figure 4. The signals are measured at different streamwise locations (upstream, inside, and downstream of the shock motion region) with a distance of $2\delta$ away from the wall along the centerline of the computational domain. In Figure 4(a), the mass-flux signal measured inside the region of shock motion oscillates between those measured upstream and downstream, indicating that the shock is moving upstream and downstream of that point. The premultiplied energy spectra plotted in Figure 4(b) show that the characteristic low frequency range is between $0.007 - 0.013U_\infty/\delta$, which is roughly the same as that given by the wall-pressure signals in Figure 2(b).

Figure 5 plots normalized iso-surfaces of $|\nabla p|$ for four consecutive instantaneous flow fields. The structures in the incoming boundary layer and the shock are seen. Two structures are highlighted in Figures 5(a) and 5(c). For an adiabatic wall, as in the DNS, these structures contain low-density, low-speed fluid. As these structures pass through the shock, the shock curves upstream, resulting in spanwise wrinkling of the shock as shown in Figures 5(b) and (d). From the data animation, the characteristic frequency of spanwise wrinkling is $O(U_\infty/\delta)$.

To analyze the unsteadiness, we introduce two definitions for the averaged shock location. First, the spanwise-mean
location, $SK_{am}$, in which the instantaneous location is defined as the point where the pressure rises to $1.3 p_{\infty}$ in the streamwise direction. Thus, $SK_{am}$ is a function of time and $z_0$. Second, the absolute mean shock location, $SK_{m}$, which is computed by spanwise and temporal averaging the instantaneous shock location. In turn, $SK_{m}$ is only a function of $z_0$. Figure 3 (b) and (d) show $SK_{am}$ and $SK_{m}$ locations.

The correlation with time lag between the pressure at $SK_{am}$ and the mass flux in the undisturbed incoming boundary layer (5δ upstream of the ramp corner) is plotted in Figure 6(a). Using $SK_{am}$ in the correlation removes the effect of the streamwise motion. The local correlation is computed first using data on spanwise planes and then the local correlations are spanwise averaged. The signals are sampled at $z_0 = 0.7δ$ since the shock is well defined there. A peak of the correlation is observed at $τ = −3.3δ/U_\infty$ (i.e., events are separated about 3δ) with a magnitude of about 0.25. The “enhanced” correlation is also plotted in the same figure, where the contribution to the correlation is only computed whenever the difference between the instantaneous shock location and $SK_{am}$ is greater than 0.15δ (or 1.5 standard deviations). In other words, only strong events are accounted for. The enhanced correlation has a similar shape to the regular correlation. It peaks at the same location with a greater magnitude, indicating that the correlation is mainly influenced by strong events. Thus, the spanwise wrinkling is related to low momentum fluid.

Figure 6(b) plots the correlation between pressure at the absolute mean shock location, $SK_{am}$, and the mass flux in the undisturbed incoming boundary layer. For the regular correlation, a peak is observed at the same location as in Figure 6(a), but with a much smaller magnitude. The enhanced correlation is also computed, using data only when the instantaneous shock location deviates from $SK_{m}$ more than 0.3δ (or 1.5 standard deviations). Again, the enhanced correlation peaks at the same location, however, the magnitude observed is still much smaller than those in Figure 6(a). Measuring the mass flux of the incoming boundary layer in the logarithmic region, where the superstructures are best identified, gives equally low correlation values. Thus, the streamwise shock motion is not significantly affected by low momentum structures in the incoming boundary layer.

Figure 6(c) plots the correlation between pressure at $SK_{am}$ and the mass flux at $(x = −5δ, y, z_0 = 0.7δ)$ and pressure at $(SK_{am}, y, z_0 = 0.7δ)$ and Figure 6(d) plots the correlation between pressure at $(SK_{am}, y, z_0 = 0.7δ)$ and pressure at $(SK_{am}, y, z_0 = 0.7δ)$. From Wu & Martin (2007b).

Computing the correlations in Figure 6 without spanwise averaging gives the same result except that the correlation curve is not as smooth due to the lesser number of samples.

Conditional statistics on the incoming boundary layer have been calculated, conditionally based on the shock being upstream or downstream of the absolute mean location. No significant difference is found in these properties. The conditionally averaged mean profiles and boundary layer parameters (Table 1) are nearly identical with very small difference (consistently less than 3%). This is in agreement with the experiments of Beresh et al. (2002) for a 25° compression ramp with $M = 5$, where the difference in the conditionally averaged mean velocity was roughly 2%.

UNSTEADINESS OF THE SEPARATION BUBBLE

The separation and reattachment points (denoted by $S$ and $R$, respectively) are defined using a $C_f = 0$ criteria. Figure 7 (a) plots the time evolution of the spanwise-mean separation point $Sm$ and the reattachment point $Rm$. The spectra for these signals also exhibit a low frequency component of about 0.01 $U_\infty/δ$. The shock foot is related to the separation point because the flow turns first near the separation bubble. Thus, we expect a strong correlation between $Sm$ and $SK_{am}$. Figure 7 (b) plots the correlation for the spanwise-mean separation point $Sm$ and $SK_{am}$ at $z_0 = 2δ$. The correlation peak is about 0.85 with a time lag of about 7δ/U_\infty. Notice the time interval between each data point in Figure 7 (b) is about 3δ/U_\infty, therefore the peak location has ±3δ/U_\infty uncertainty. This uncertainty also applies for all of the following correlations with time lag. Gana-pathisubramani et al. (2007) correlated the instantaneous separation point $S$ (defined using a velocity threshold criteria) and streamwise-averaged values of streamwise velocity in the incoming boundary layer at $z_0 = 0.2δ$. The same analysis performed here yields a correlation of about 0.5, which is similar to the value 0.4 found by Gana-pathisubramani et al. (2007). Figure 8a plots the profile for the correlation between the instantaneous separation point using the $C_f = 0$ definition and streamwise-averaged values of $p_{\infty}$, where the streamwise averaging is performed from the separation point to the inlet. Using the $C_f = 0$ criteria, the correlation factor at $z_0 = 0.2$ is 0.23. Thus, the use of the actual definition of the separation point decreases the correlation between the separation point and the streamwise-averaged $u$ significantly.

Figure 8b plots two correlations, the correlation between $Sm$ and $Rm$ and the correlation between the shock location $SK_{am}$ and $Rm$.
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the Strouhal number of the low frequency does not seem to have a significant dependence on Mach number, suggesting that acoustic resonance may not cause the low-frequency shock motion. It is interesting to point out that in cavity flows, there are two modes observed (Gharib & Roshko, 1987; Rowley et al., 2002): the shear-layer mode and the wake mode. In this case, acoustic resonance is responsible for the generation of the shear-layer mode, while the wake mode is purely hydrodynamic. Moreover, the wake mode corresponds to larger scale and lower frequency motions compared to the shear-layer mode. Providing that there are some similarities in compression ramp interactions and cavity flows in that they all have a shear layer formed above a separated region, we suggest that the mechanism of the low-frequency shock unsteadiness may resemble that of the generation of the wake mode in cavity flows. In other separated flows, for example, flow passing a backward-facing step, low frequency fluctuations have also been indicated (e.g. Simpson, 1989), while the driving mechanisms are still not fully understood.

DNS data animations show that the size (including the length and height) of the separation bubble changes significant with a low frequency that is comparable to that of the low frequency shock motion. Figure 9 plots six consecutive times in the DNS with time intervals of about $\delta/U_\infty$, showing the break down of the separation bubble indicated by streamlines. Flow quantities are averaged in the spanwise direction to get a clear picture. Contours of pressure gradient are also plotted to show the shock location. From frame (c) to (f), fluid bursts outside the separation bubble, causing the bubble to shrink. The shock then moves downstream at a later time (not seen in the figure). To show how the separation bubble changes with time, the mass and the area of reverse flow region inside the separation bubble are plotted in Figure 10(a). The reverse flow region is defined as regions in which $u$ is negative, where $u$ is spanwise averaged. It is observed that the mass inside the reverse flow region has an

d(ISU) 2006) saw correlation between the instantaneous separation point and streamwise averaged values of $\phi u$ and (b) correlation between the separation and reattachment point and the shock location at $z_a = 2\delta$. From Wu & Martin (2007b).

DISCUSSION

From the above analyses, the local spanwise wrinkling shock motion is shown to correlate with low momentum fluid in the incoming boundary layer, which is consistent with what Wu & Miles (2001) found in a compression ramp interaction using high speed visualization techniques. However, the spanwise wrinkling is a smaller scale, local unsteadiness compared with the streamwise shock motion. The small correlation between the low momentum fluid in the incoming boundary layer and the separation point found in the DNS implies that these low momentum structures might have a relatively minor contribution to the shock unsteadiness. The negative time lag in the correlation between the shock location and reattachment point suggests that the separation region may play an important role in driving the low frequency shock unsteadiness, as seen experimentally by Thomas et al. (1994). The fact that the Strouhal number of the low frequency shock motion defined using the separation length lies in the experimental range (DuSSauge et al. (2006)) is also supportive for this argument. Pirossoli & Grasso (2006) performed a DNS of a reflected shock interaction and proposed that the shock unsteadiness was sustained by an acoustic resonance mechanism that is responsible for generating tones in cavity flows. However, the low-frequency shock motion may not be captured in their DNS due to the fact that the lowest Strouhal number reported is between 0.09 and 0.24, which is above the range 0.02-0.05 found in experiments. According to Dussauge et al. (2006), the Strouhal number of the low frequency motion does not seem to have a significant dependence on Mach number, suggesting that acoustic resonance may not cause the low-frequency shock motion. It is interesting to point out that in cavity flows, there are two modes observed (Gharib & Roshko, 1987; Rowley et al., 2002): the shear-layer mode and the wake mode. In this case, acoustic resonance is responsible for the generation of the shear-layer mode, while the wake mode is purely hydrodynamic. Moreover, the wake mode corresponds to larger scale and lower frequency motions compared to the shear-layer mode. Providing that there are some similarities in compression ramp interactions and cavity flows in that they all have a shear layer formed above a separated region, we suggest that the mechanism of the low-frequency shock unsteadiness may resemble that of the generation of the wake mode in cavity flows. In other separated flows, for example, flow passing a backward-facing step, low frequency fluctuations have also been indicated (e.g. Simpson, 1989), while the driving mechanisms are still not fully understood.

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intermittent character, just like the momentum signal inside the shock motion region show in Figure 4. Figure 10(b) plots the correlation of the mass signal with the spanwise averaged shock location $S K_{\text{am}}$ at $z = 26$ and with the wall pressure difference $\Delta P = P_w(x = 1\delta) - P_w(x = -2\delta)$. From Wu & Martin (2007b).

Based on the above observations, it is hypothesized that one of the mechanisms driving the low-frequency shock motion can be described as a feedback loop between the separation bubble, the separated shear layer and the shock system, which has some similarities with the cause of the low-frequency “flapping motion” in backward-facing step flows described by Eaton & Johnston (1981). That is, the balance between shear layer entrainment from the separation bubble and injection near the reattachment point is perturbed. If the injection is greater, the separation bubble grows in size and causes the reattachment point to move downstream and the separation point to move upstream. The motion of the separation point causes the shock to move with it. As the shock moves upstream, the pressure gradient in the separation region decreases due to the enlargement of the separation bubble and decreasing of streamline curvature. The decreasing pressure gradient reduces the entrainment of fluid into the separation bubble. In turn, the separation bubble becomes unstable and breaks down. When this happens, fluid bursts outside the bubble and the separation region shrinks fairly rapidly, causing the shock to move downstream at a later time. Similarly, when the shock moves to a downstream location, the overall pressure gradient in the separation region increases, which enhances entrainment of fluid into the separation bubble, causing the bubble to grow. Thus, the low frequency shock motion is closely related to the time scale associated with the growth and burst of the separation bubble. Assuming that this time scale is determined by the length of the separation bubble $L$ and the characteristic speed of the reverse flow $U_R$, the dimensionless shock frequency $S t_R = f L/U_R$ can be computed. Using the maximum of the time-averaged reverse flow speed in the separation bubble, $0.055 U_\infty$, to represent $U_R$, the dimensionless frequency $S t_R$ in the DNS is around unity (about 0.8).

**CONCLUSION**

Wall-pressure and separation point signals indicate low-frequency motions in DNS data of a 24° compression ramp. Analyses show that the shock motion is characterized by a low-frequency, large-amplitude streamwise motion with characteristic frequency of about $0.013 U_\infty/\delta$, and a relative smaller-amplitude, high-frequency $O(U_\infty/\delta)$ spanwise wrinkling. The mass flux in the incoming boundary layer is correlated with the high-frequency spanwise wrinkling motion. Conditional statistics indicate no significant difference in the mean properties of the incoming boundary layer when the shock is upstream/downstream.

The location of the separation point is highly correlated with shock location with a time lag of about $7\delta/U_\infty$. A small correlation is found between the low momentum structures in the incoming boundary layer and the separation point, indicating the influence of the superstructures on the shock motion may be minor. However, it is found that both the shock motion and the separation point motion are correlated with and lag the motion of the reattachment point, suggesting that the downstream flow plays an important role in driving the low frequency shock motion. A model that is described as a feedback loop between the separation bubble, the separated shear layer, and the shock system is proposed to explain the low frequency shock motion. Using the length of the separation bubble and the characteristic reverse flow speed (e.g. the maximum of the mean reverse flow speed), the Strouhal number of the low frequency shock motion is around unity.

We acknowledge insightful discussions with Prof. A.J. Smits and the support from the Air Force Office of Scientific Research under grant no. AF/9550-06-1-0323.

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