TURBULENCE IN SUPERSONIC AND HYPERSONIC BOUNDARY LAYERS

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Abstract: A summary is given of the behavior of turbulent boundary layers in supersonic and hypersonic flow where the effects of compressibility have a direct influence on the turbulence. Experimental and DNS results are presented and compared.

Key words: Turbulence, supersonic, hypersonic, shocks, shocklets, boundary layer

1. INTRODUCTION

In 1951, van Driest published a seminal paper entitled Turbulent boundary layer in compressible fluids [1] that founded the study of turbulent boundary layers at high speed. Four years later, he was present at the Braunschweig meeting on Fifty years of boundary layer research [2], and reported on the status of that fledgling field. The fifty years were, of course, counted from the date Prandtl delivered his famous lecture on flows with very small friction at the Third Mathematical Congress in Heidelberg in 1904, thereby establishing for the first time the concept of the boundary layer [3]. The occasion of the present symposium, another fifty years on from the Braunschweig meeting, offers an excellent opportunity to consider where we currently stand, and what prospects there are for future progress in understanding the behavior of turbulent boundary layers in supersonic and hypersonic flow, that is, where the effects of compressibility have a direct influence on the turbulence.

The most important parameter in the description of incompressible turbulent boundary layer behavior is undoubtedly the Reynolds number. For compressible flows, the Mach number becomes a further scaling parameter. Within the boundary layer, the flow is supersonic in the outer layer and subsonic near the wall, although the sonic line is located very close to the wall at high Mach number. A temperature gradient develops across the boundary layer due to the conversion of kinetic energy to heat as the flow velocity decreases. In fact, the static-temperature variation can be very large even in an adiabatic flow, resulting in a low-density, high-viscosity region near the wall. In turn, this leads to a skewed mass-flux profile, a thicker boundary layer, and a region in which viscous effects are somewhat more important than at an equivalent Reynolds number in subsonic flow.

The temperature variations across the layer also cause the fluid properties to vary. For example, for an air flow with a freestream Mach number of 3 on an adiabatic wall, the density varies across the boundary layer by a factor of about 5, while the kinematic viscosity varies by a factor of about 17 [4].

Intuitively, one would expect to see significant dynamical differences between subsonic and supersonic boundary layers. However, it appears that many of the apparent differences can be explained by accounting for the fluid-property variations that accompany the temperature variation. This suggests a rather passive role for the density differences in these flows, most clearly expressed by Morkovin's hypothesis, which states that the dynamics of a compressible boundary layer are expected to follow the incompressible behavior closely, as long as the Mach number associated with the fluctuations remains small. We interpret this to mean that the fluctuating Mach number, M', must remain small, where M' is the rms perturbation of the instantaneous Mach number from its mean value, taking into account the variations in velocity and sound speed with time. If M' approaches unity at any point, we expect direct compressibility effects such as local shocklets and pressure fluctuations to become important. If we take M' = 0.3 as the point where compressibility effects could start to become important for the turbulence behavior, Smits & Dussauge [4] estimated that for zero-pressure-gradient adiabatic boundary layers at moderately high Reynolds numbers this point would be reached with a freestream Mach number of about 4 or 5 (see Figure 1a).

In fact, we find from experiments performed by Baumgartner *et al.* [5] that compressibility effects on turbulence in zero pressure gradient flows are weak, even at M = 8. The only significant effect seems to be on the integral length scale, which decreases significantly with Mach number (see Figures 1b and 2). All other statistical measures, such as the Reynolds stresses and the higher order moments, do not show any obvious influence of Mach number, as long as they are scaled (when appropriate) by the local density rather than the density at a fixed point [4]. This scaling is called the

Morkovin scaling Other quantities, such as the intermittency, also do not appear to be a function of Mach number, despite earlier evidence to the contrary (for example, compare the intermittency data in [5] with that in [6]).

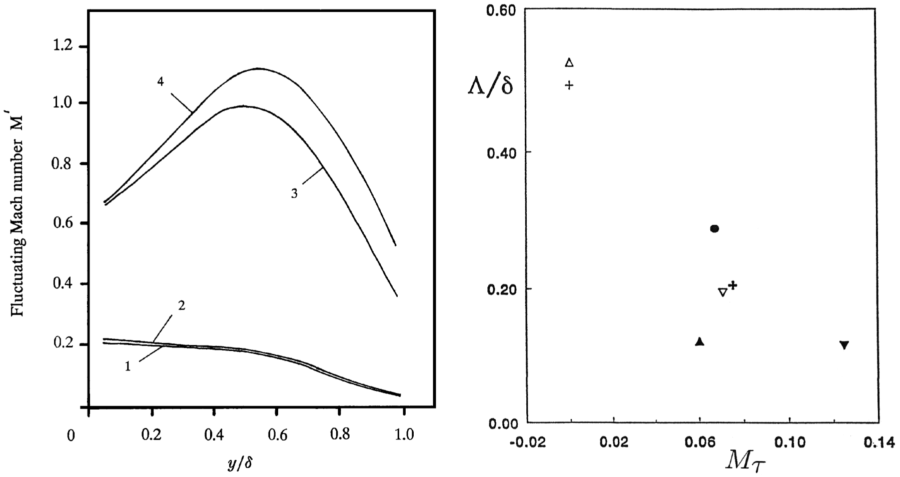

Figure 1. (a): Fluctuating Mach number distributions (estimated). Flow 1: M = 2.32; Flow 2: M = 2.87; Flow 3: M = 7.2; Flow 4: M = 9.4. (b) Integral scale as a function of friction Mach number in boundary layer flow (by experiment). Figures from [4], where original references are given.

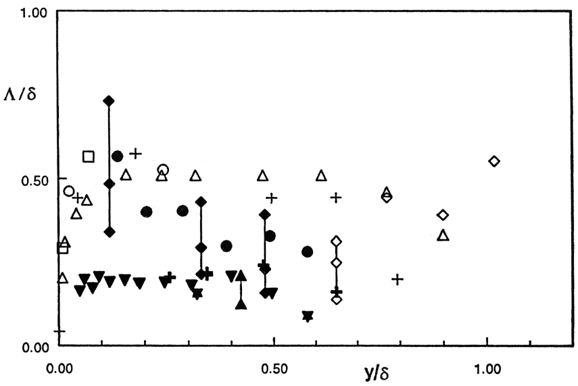

Figure 2. Integral scale as a function of freestream Mach number in boundary layer flow (by experiment). Open symbols: subsonic flow. Closed symbols: supersonic and hypersonic flow. Figure from [4], where original references are given.

Nevertheless, the role of intense Mach number fluctuations is interesting from a fundamental point of view, as well as perhaps playing a part in decreasing the integral length scale as the Mach number increases. In this respect, shocklets (local shocks occurring within the boundary layer due to supersonic relative motions) may be important. Although shocklets have been visualized in hypersonic boundary layers [5], only Direct Numerical Simulations (DNS) can be used to examine their structure and their possible role in rescaling turbulent motions. Many other open questions exist, regarding, for example, the level of the pressure fluctuations within the boundary layer, the velocity-temperature correlation, the accuracy of Reynolds analogies, the kinetic energy budget, and, most intriguingly, the structure of the coherent motions. Most of these quantities cannot be measured accurately, or even at all, and DNS, appropriately validated, may be crucial in making further progress.

Direct numerical simulations of boundary layers in supersonic flow have only become available recently. The first results were reported in 2000 by Adams [7], who studied the flow over a compression ramp at M = 3 and a Reynolds number based on momentum thickness $Re_q = 1685$. The impact of Adam s work was somewhat limited by the low value of his Reynolds number, in that there were no experiments available for comparison with his results at that time. More recently, Martin [8-9] reported DNS of hypersonic turbulent boundary layers at higher Reynolds numbers, allowing comparison with data taken at IMST in France [11-14]. Having established the plausibility of her computations, Martin studied the behavior of the boundary layer as the Mach number varied from 3 to 8 while keeping the Reynolds number in wall units approximately constant. The purpose of the present contribution is to discuss these results as well as recent experimental data, and consider how they may change our understanding of high-speed turbulent boundary layers.

2. COMPARISON OF DNS AND EXPERIMENT

Martin s computational data are available for the conditions given in Table 1 [9]. Typical values for grid resolution and domain size are $L_x/\delta = 7.9$, $L_y/\delta = 2.0$, $L_z/\delta = 15.4$, $\Delta x^+ = 7.6$, $\Delta y^+ = 2.8$, $N_x = 384$, $N_y = 256$, $N_z = 110$ (Case M4). Here x, y and z are the streamwise, wall-normal, and spanwise directions, respectively. The wall was isothermal, and the effects of varying the wall temperature were assessed. Although the results were presented for a time-developing layer, and Martin showed that only small differences existed between that temporal and spatially-developing layers. Martin also demonstrated very good agreement with experimental data for Case M2, establishing a high degree of confidence in the entire data set.

The DNS over this Mach number range (3 to 8) support many observations gleaned from experiment. For example, the mean velocity profiles transformed according to van Driest collapse with the usual scaling using inner and outer variables. Also, the Reynolds stresses collapse using

Morkovin's scaling at about the same level of accuracy as seen in experiment, the intermittency profile shows little influence of Mach number (although there is an unexplained peak near the wall that exceeds a value of one, see Figure 3a), and the temperature/velocity correlation R_{uT} is almost independent of Mach number and constant at about 0.7 for most of the layer (Figure 3b), although that is a little lower than the generally accepted experimental value of 0.8.

Case	M_δ	Re_θ	δ^+
M2	2.32	4452	745
M3	2.98	2390	325
M4	3.98	3944	368
M5	4.97	6225	382
M6	5.95	8433	396
M7	6.95	10160	414
M8	7.95	13060	430

Table 1. Boundary layer parameters for DNS computations Cases M2 to M8 [9].

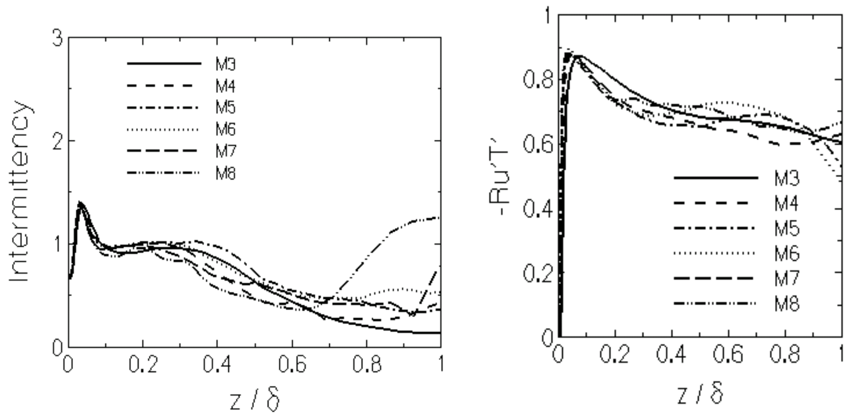

Figure 3. DNS results from Martin [9]. Flow conditions given in Table 1. (a) Intermittency profiles determined by 3/flatness; (b) Temperature/velocity correlation.

For the fluctuating pressure levels, with the wall temperature set approximately equal to its adiabatic value, the fluctuations increased from about 2% in the freestream to 4 or 5% near the wall, with only a weak Mach number dependence (Figure 4a). Experiments in a M = 1.8 flow showed a similar increase but only from about 0.3% in the freestream to 1% at the wall [4]. It seems the DNS results are too high, which may be the result of insufficient averaging. However, the computations reveal that at Mach 5 decreasing the wall temperature by a factor three from its adiabatic value increases the level of the pressure fluctuations by about 75%, which is a new result if confirmed in subsequent work.

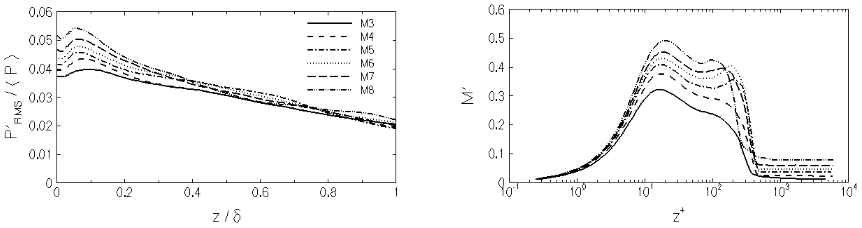

Figure 4. DNS results from Martin [9]. Flow conditions given in Table 1. (a) Pressure fluctuation distributions; (b) Fluctuating Mach number distributions.

Interestingly, the fluctuating Mach numbers do not indicate the high values expected from the estimates shown in Figure 1a. For example, the DNS results indicate a maximum value of about 0.5 at Mach 8, compared to the estimated value of 1.0 at the same Mach number (Figure 4b). There is apparently little sensitivity with wall temperature, a result that was expected from previous work, but the factor of two difference between the estimate made by [4] and the DNS results is potentially very important, and may help to explain why the Mach number effects detected by experiment [5] are generally rather small.

Despite the small values of M , shocklets were observed in the DNS. Figure 5a shows instantaneous contours of pressure in a Mach 4 turbulent boundary layer from Martin [8]. Shocklets were also observed by Baumgartner *et al.* [5] in Filtered Rayleigh Scattering (FRS) images taken in a Mach 8 boundary layer. In the case of DNS, the presence of shocklets is found by the large magnitude of the correlation between gradients of the divergence (from positive to negative) with large pressure gradients (of opposite sign). Verifying that the Rankine-Hugoniot conditions are met along the instantaneous streamline further corroborates the presence of shocklets. The window in the figure shows the location of a shocklet, which is a small-length, small-time scale shock. Further study of the role of shocklets in the turbulence dynamics are in progress at Princeton.

With respect to the integral length scale, Figure 5b shows the DNS results, demonstrating, as expected, a decrease in size as the Mach number increases. As the freestream Mach number increases from 3 to 8, the value decreases about 50%, which is in good accord with the experiment (Figure 2), given the rather scattered nature of the data.

Under the conditions of negligible fluctuations in total enthalpy, the one-dimensional energy equation gives:

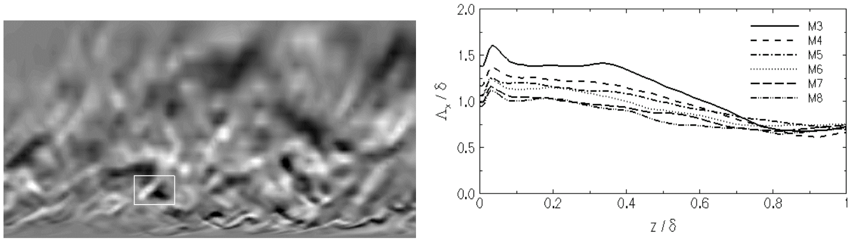

Figure 5. (a) Divergence of velocity for a Mach 4 turbulent boundary layer in a streamwise plane. DNS results from [8], flow is from left to right. (b) Integral length scale computed from DNS data [9].

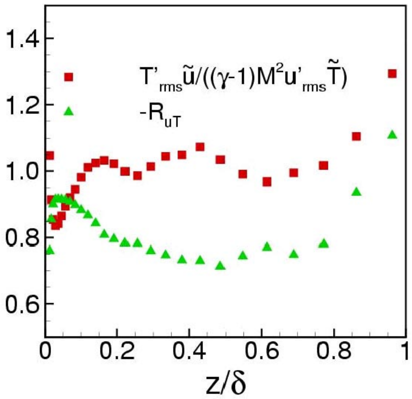

Figure 6. Test of the Strong Reynolds Analogy for a Mach 3 turbulent boundary layer, $\mathrm{Re}_\theta = 2{,}400$ ($\delta^+ = 400$). DNS results, adapted from Wu & Martin [10].

$$\frac{\sqrt{\overline{T'^2}}}{\overline{T}} = (\gamma-1)M^2\frac{\sqrt{\overline{u'^2}}}{\overline{U}} \tag{1}$$

and

$$R_{uT} = -\frac{\overline{u'T'}}{\sqrt{\overline{u'^2}}\sqrt{\overline{T'^2}}} = -1 \tag{2}$$

These relations are often called the Strong Reynolds Analogy (SRA), and they are commonly used in experiment to relate the temperature and velocity fluctuations where only one of the two quantities is known [4]. The DNS results for the temperature velocity correlation R_{uT} as a function of Mach number were shown earlier in Figure 3b. For a Mach 3 boundary layer with Re_q = 2400, results from additional DNS confirm the experimental observation that the correlation level varies between 0.7 and 0.8 over most of the layer (Figure 6). This value is smaller than that given by Equation 2, but somewhat surprisingly the comparison with Equation 1 is almost perfect, except for a small region near the wall where viscous effects are undoubtedly important. The two observations taken together suggest a phase difference between the velocity and temperature signals, as first discussed by Smith & Smits [15]. Further analysis of the DNS may provide a more definitive answer to this suggestion.

One of the most direct comparisons between DNS and experimental data can be made visually. In Figure 7, we show snapshots in a streamwise plane, taken from two nominally identical flows, one experimental and one DNS. The qualitative similarities are obvious. More detailed analysis of the experimental images will provide quantitative data on streamwise length scales, structure angles, and intermittency. A preliminary result on the intermittency profile is given in Figure 8.

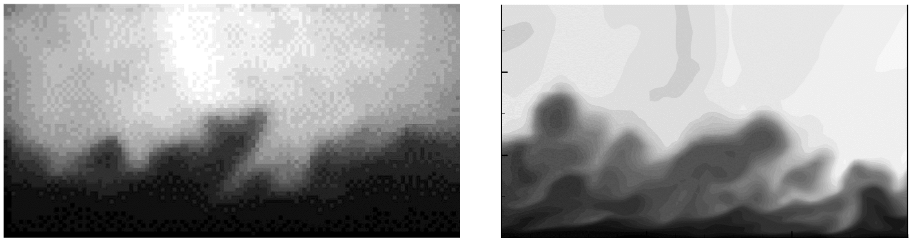

Figure 7. Mach 3 boundary layer, flow is from left to right. (a) Experimental FRS data, $Re_\theta = 2,397$. Bookey & Wyckham, private communication. (b) Density contours computed from DNS data, $Re_\theta = 2,400$ ($\delta^+ = 400$). Martin, private communication.

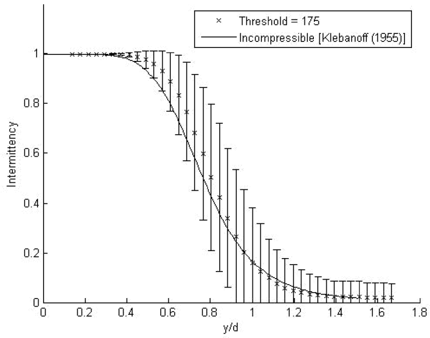

Figure 8. Intermittency profile from experimental FRS data, same boundary layer as in Figure 7. Bookey, private communication.

3. CONCLUDING REMARKS

DNS of turbulent boundary layers at supersonic and hypersonic speeds have only started to appear in the literature within the last four years.

Current results are still limited to relatively low Reynolds numbers, but the Reynolds numbers are already within the range where direct comparisons with experiment are possible. The data from the group headed by Dussauge at IUSTI in Marseille (formerly IMST) has been particularly valuable in this regard. Current efforts at Princeton to obtain experimental data under the same flow conditions as the DNS by Martin are helping to further increase our confidence in the computations.

Despite the limitations on Reynolds number, the detailed information on turbulence statistics and turbulence structure provided by DNS is already enriching our understanding of the behavior of turbulence in supersonic and hypersonic flows. Further analysis of the computations may well lead us to understand more fully the role that compressibility plays in turbulent boundary layers. For example, the indications that fluctuating Mach numbers in high Mach number flows are considerably lower than formerly believed leads to two possible conclusions. First, the analysis used to form the estimates given in Figure 1a may not have been correct. These estimates made use of Morkovin's hypothesis and the Strong Reynolds Analogy in a very simple way, and although the DNS results generally support these scalings, it appears that the sum of the parts does not add up to the effects observed. Second, we may also conclude that the direct effects of compressibility on wall-bounded flows are even smaller than formerly believed, implying also that shocklets do not have a strong influence on the dynamics of turbulence even at Mach 8. Nevertheless, we see, for example, a rapid change in integral length scale with increasing Mach number in experiments and in DNS. The underlying cause of this phenomenon is still to be found, and we expect to make dramatic progress in the next few years as more DNS data become available.

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