DNS of a Large-Domain, Mach 3 Turbulent Boundary Layer: Turbulence Structure

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We introduce a Mach 2.9, $Re_{\tau} = 640$, spatially developing direct numerical simulation on a very large domain which is about 60 incoming boundary layer thicknesses long in the streamwise direction. Using the rescaling technique of Xu & Martin,¹ with the rescaling plane taken near the exit, at about 57 incoming boundary layer thicknesses (just under 0.4 meters downstream of the inlet), we obtain a clean inflow with a good sampling of even the largest scales. We show that the simulation can be run for long times $\geq 450\delta/U_{\infty}$ without the forcing of artificial acoustic modes in the free stream. We proceed to examine the turbulence structure through spectral analysis and filtered instantaneous flow fields. Special attention is paid to the largest structures, with turbulence modeling, especially aspects unique to compressible boundary layers, in mind.

I. Introduction

The existence of coherent vorticle motions, or coherent structures, in boundary layers is now widely accepted. Theodorsen² postulated the existence of the hairpin vortex, a simple flow structure that explains the formation of low-speed streamwise streaks and the ejection of near-wall low-momentum fluid into higher-momentum regions farther from the wall; see Figure 1(a) for a conceptual drawing. Head & Bandyopadhyay³ provided experimental evidence of the streamwise stacking of individual hairpin vortices into larger structures, packets, whose heads describe an envelope inclined at a 15° to 20° downstream angle with respect to the wall. More recently, Adrian, Meinhart, & Tomkins⁴ proposed a hairpin packet model, where the hairpins in a packet align in the streamwise direction as observed by Head and Bandyopadhyay. In this model, packets enclose regions of low momentum induced by their heads and counter-rotating legs, and align themselves in the streamwise direction giving rise to the low-momentum, very large-scale motions (VLSM) observed by Jiménez⁵ and Kim & Adrian,⁶ see Figure 1(b). These long low-momentum streaks in the logarithmic region have also been observed by Hutchins & Marusic,⁷ who call them "super structures." Recently Marusic and coworkers have proposed a model for the near wall streamwise fluctuations based on the observations studying superstructures.⁸ This model displays remarkable Reynolds number independence and can accurately reproduce the near wall fluctuations, spectra and high order moments using information only obtained from the middle of the logarithmic region. In this model the presence of "super structures" influences the near wall fluctuations through two mechanisms: amplitude modulation of the high frequencies and superposition of the low frequency components. Figure 2^8 is a depiction of the low momentum streaks and how they interact with the near wall region.

The current study of the turbulence structure in boundary layers has been confined largely to the subsonic flow regime (Tomkins & Adrian;⁹ del Álamo & Jiménez;¹⁰ Ganapathisubramani, Longmire & Marusic;¹¹ del Álamo *et al.*;¹² del Álamo *et al.*;¹³ Guala, Hommena & Adrian;¹⁴ Hambleton, Hutchins & Marusic;¹⁵ Flores *et al.*;¹⁶ Balakumar & Adrian;¹⁷ Hutchins & Marusic⁷ and Mathis, Hutchins & Marusic,¹⁸ for example). The study of supersonic and hypersonic turbulent boundary layers has been primarily restricted to simple statistical analysis, due to the lack of detailed flow field data and the difficulty of performing experiments and simulations in this regime. Fernholz & Finley;^{19,20} Spina, Donovan & Smits;²¹ Smits & Wood;²² Fernholz *et al.*;²³ and Smits & Dussauge,²⁴ give reviews including the effects of pressure gradient, streamline curvature

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and the interaction with shock waves in high-speed turbulent boundary layers. These descriptions are statistical and some include qualitative flow visualizations. The only structure information such as convection velocity, angle, and length scale, has been obtained from space-time correlations (see, for example, Smits $et \ al.,^{25}$ Spina, Donovan & Smits,²⁶ and Smits & Dussauge²⁴). The results indicate changes in structure properties with both Mach and Reynolds numbers, such as a decrease in structure length with increasing Mach number.

Recent advances in numerical and experimental techniques allow for detailed four-dimensional, in space and time, flow field data acquisition. Direct numerical simulations make possible the computation of turbulent boundary layers at supersonic and hypersonic Mach numbers (Guarini *et al.*;²⁷ Martin;^{28, 29} Pirozzoli, Grasso & Gatski;³⁰ Xu & Martin;¹ Ringuette, Wu & Martin³¹) as well as the interaction of turbulence with strong, unsteady shock waves (Adams;³² Pirozzoli & Grasso;³³ Wu & Martin;³⁴ Wu & Martin;³⁵ Taylor, Grube & Martin;³⁶ Grube, Taylor & Martin³⁷). Advanced particle-image-velocimetry techniques allow the temporal and spatial characterization of experimental supersonic turbulent boundary layers and shock boundary layer interactions (Elsinga *et al.*;³⁸Schrijer, Scarano & van Oudheusden;³⁹ van Oudheusden;⁴⁰ Humble, Scarano & van Oudheusden⁴¹). Both numerical^{31,34} and experimental^{38,42,43} data at supersonic Mach numbers have shown evidence of VLSM.

In this paper, we present the DNS data of a turbulent boundary layer spatially developing over a length of 57 incoming boundary layer thicknesses, i.e. nearly 0.4 meters. Data are gathered over 450 δ/U_{∞} time scales. The flow configuration and computational setup are described in Section II. The quality of the data is presented in Section III. Turbulence spectral and large-scale turbulence structure analysis are given in Sections IV and V, respectively. Finally, conclusions are given in Section VI.

II. Flow Configuration and Computational Details

The flow configuration considered in the present work is a spatially developing, zero pressure-gradient flat plate boundary layer over a smooth wall at Mach 2.9. The domain is quite large, especially in the streamwise direction, with a physical length of just under 0.4 meters (381 mm). Over this distance the boundary layer grows considerably with an initial thickness of just under 7.25 mm at the inlet to just over 13.5mm at the outlet. The momentum thickness Reynolds number, Re_{θ} , varies from inlet to outlet changing from just over 3000 to nearly 5000 at the exit. ($Re_{\theta} = U_{\infty}\theta/\nu_{\infty}$ where U_{∞} denotes the freestream velocity, ν_{∞} the free stream kinematic viscosity, and θ the momentum thickness). For a diagram of the computational domain (truncated in the wall-normal, z, direction) see Figure 3. A table outlining the grid dimensions and resolution is given below in table 1. Here we use constant spacing in the streamwise, x, and spanwise, y, directions, and geometric grid stretching in the wall-normal, z, direction, where $z(k) = z_2(\alpha^{k-1} - 1)/(\alpha - 1)$. The grid spacing normalized by inner units is denoted as Δ^+ .

Direction:	streamwise (x)	spanwise (y)	wall-normal (z)	size
L/δ_0 :	59.5	4.96	9.09	70ml
grid points	2484	546	110	≈ 150 Million
Δ^+	9.24	3.51	0.263	

Table 1. Grid resolution and domain size for the DNS. The grid is equispaced in the streamwise and spanwise directions and uses a geometric stretching in the wall normal direction. Here, $\delta_0 = 6.41$ mm and $\Delta_z^+ = z_2 - z_1$

The numerical scheme used here is identical to that used and validated by Martin^{28, 29} across multiple Mach numbers and wall temperature conditions. We use a 4th-order accurate, weighted essentially non-oscillatory (WENO) scheme, which has been linearly optimized.⁴⁴ The optimization process reduces the dissipation and dispersion in smooth regions by adding an additional candidate stencil and adjusting the optimal stencil weights, resulting in the linear portion of the scheme being symmetric and bandwidth resolving efficiency optimized. For the spatial discretization of the viscous fluxes, 4th-order accurate central differencing is used, and time-integration is performed with a 3rd-order accurate, low-storage, explicit Runge-Kutta algorithm. The flow is initialized following the method of Martin.²⁹

The boundary conditions are as follows. At the wall, a no-slip, isothermal boundary condition is used. The prescribed wall temperature is 307K, which is close to the adiabatic wall temperature. At the top and outlet boundaries, a supersonic exit boundary condition is used. The flow has a free stream Mach number, M_{∞} , of 2.91, a free stream velocity, U_{∞} , of 610 m/s, a free stream temperature, T_{∞} , of 109 K and the density in the free stream, ρ_{∞} , is 0.076 kg/m³. Periodic boundary conditions are implemented in the spanwise direction.

The inflow boundary condition is provided by the rescaling technique outlined by Xu & Martin.¹ This method has the advantage of being relatively simple to implement, all boundary variables are appropriately specified, and its action is well understood; we do not use any complicated reflection or shifting of the recycling plane. In our experience, when the DNS is run for long times (of the order of several hundreds of δ/U_{∞}), and a rescaling length typical of those commonly found in the literature is used, say 8δ , it becomes apparent that the rescaling forces an acoustic mode in the freestream and can be clearly seen in time series spectra. This forcing is a narrow band phenomena at frequencies of U_c/λ_0 and higher harmonics. Here U_c is the applicable convection velocity, the local mean for instance, and λ_0 is the distance between the inflow plane and the recycling plane.

In addition to unphysical acoustic modes appearing in the free stream, the physics of the larger coherent structures may be artificially altered if the rescaling length is not sufficiently large. Large scale structures which convect faster, and change more slowly, are artificially reintroduced before they can sufficiently evolve. Extending the analysis of Simens et al.⁴⁵ by invoking Morkovin's hypothesis for compressible boundary layers, we may define an eddy evolution time scale for eddies of size $O(\delta)$. Such eddies have internal velocities of $O((\rho_{\infty}/\rho_{w})^{1/2}u_{\tau})$ which yields an eddy evolution time scale of $(\rho_{w}/\rho_{\infty})^{1/2}\delta/u_{\tau}$. (Under Morkovin's hypothesis the dominant effects of compressibility reduce to the effects of variable fluid properties, predominantly manifested in variable density and viscosity. For more details see Smits & Dussauge.²⁴) Over this time scale, eddies of size $O(\delta)$ convect with a speed of $O(U_{\infty})$ and will cover a distance of $U_{\infty}(\rho_w/\rho_{\infty})^{1/2}\delta/u_{\tau}$. For the conditions of the present simulation this large eddy evolution length scale works out to be $O(12\delta)$. This estimation is optimistic because, as Simens notes, the true evolution time is probably closer to twice this. Furthermore, the validity of Morkovin's scaling—especially in the outer region of the flow and at these DNS accessible Reynold's numbers—is a topic of ongoing research. Using Simens' original large eddy evolution time scale definition of δ/u_{τ} we arrive at a necessary rescaling length of at least 20 δ to enable a large eddy to evolve sufficiently before recycling it. An alternative minimum criteria might be to have a rescaling length, λ_0 be disjoint from the wavelength support of the turbulence spectra. This ensures that there is no turbulence energy at length scales longer than the rescaling length which would indicate the potential for turbulence modes to have significant self-interaction. This is an extraordinarily constraining requirement. especially at increasing Reynolds numbers, and may still be somewhat insufficient in terms of the largest turbulent motions, or "super structures." In light of this analysis, we use a rescaling length of $57\delta_0$ as seen in Figure 3 to attempt to minimize any artificial forcing from the rescaling technique. In the next section, we show that a rescaling length of 57 δ is indeed nearly disjoint from even the largest energetic turbulent length scales, and produces no detectable rescaling mode in the turbulence spectrum.

III. Assessment of the DNS

In this section we examine the DNS data in terms of the fidelity of the simulation. The DNS code used here has been extensively validated across a wide range Mach numbers and wall temperatures, including those of the present study.²⁹ We must still, however, assess the quality of the current simulation in terms of domain size, grid convergence, and quality of inflow boundary condition.

Despite a mesh of about 150 million points, the present simulation is not grid converged. While the C_f versus Re_{θ} matches the van Driest II semi-empirical predictions to within $\pm 5\%$, as can be seen in Figure 4, the Van Driest transformed velocity profile (Figure 5) indicates that the log layer additive constant is 6, well above the usual 5.2. This is indicative of excessive numerical dissipation. Despite this, we feel that the results presented in this paper should be qualitatively correct except where noted. We are currently working to attain grid convergence.

Next, we assess the domain size and inflow boundary condition. Since periodicity is being used in the spanwise direction we must ensure that the domain is sufficiently wide to contain an appropriate sample of the large structures. Below, in Figure 6, we can see that two point auto-correlations of ρu are very small at spanwise separations of half the domain thickness for all streamwise stations. As the boundary layer grows moving downstream so does the width of the turbulence structures, but even at a wall-normal height of $z/\delta = 0.8$ the correlation levels are acceptable.

Since the rescaling plane was chosen near the outflow boundary, the domain streamwise length is sufficient to hold a representative sample of the large scales. We focus on looking for any evidence of artificial forcing due to the rescaling technique, examining the wave-number pressure spectrum, ϕ_{pp} , and the streamwise velocity spectrum, ϕ_{uu} , at the rescaling station. These are plotted at various wall normal locations in premultiplied form in Figure 7(a,b). We use Taylor's hypothesis of "frozen" convection to convert a time series signal into a wave number signal, taking the convection velocity, U_c , as $\overline{U(z)}$. We do this to enable better comparisons to experiments and because it is the only effective means of extracting converged streamwise (low) wavenumber spectra. Note that wavelength, λ_x , is plotted on the abscissa, rather than wave-number, k_x . The wavelength corresponding to rescaling, λ_0 , and it's first harmonic are given as black vertical lines. As we can see below in Figure 7(a,b), we have no more energy at large wave lengths than we would expect anywhere in the boundary layer and free stream. Further more, if we examine a time series spectrogram of $\phi_{pp}(St, z)$, where $St = f \delta/U_{\infty}$ is the Strouhal number, we see negligible energy at the frequencies associated with the inflow recycling as seen in Figure 8. The convergence of this figure could be better, but the convergence is limited by the length of the simulation and the lowest frequency we wish to resolve.

An additional visual inspection, while only heuristic and qualitative, may be performed by plotting instantaneous numerical schlieren visualizations of the flow at two instances in time, t_a , and t_b separated by λ_0/U_c . (We take $U_c \approx U_{\infty}$.) Numerical schlieren is defined in equation 1.

$$NS = 0.8 \exp\left(-10 \frac{|\nabla \rho| - |\nabla \rho|_{min}}{|\nabla \rho|_{max} - |\nabla \rho|_{min}}\right) \tag{1}$$

As can be seen in Figure 9, each eddy has evolved significantly over the period of time it takes to travel from the inlet to the rescaling plane.

IV. Turbulence Spectra

In this section we discuss the spectral analysis of the flow field, all of which is performed at the rescaling plane, where the von Kármán number, Re_{τ} , is 640. The form of the turbulence spectra for compressible boundary layers is essentially still unverified, save for a few experiments and simulations, although theoretical arguments for incompressible boundary layers can be extended to compressible ones via Morkovin's hypothesis.²⁴ One such experimental study in which turbulence spectra are measured is that of Elsinga *et al.*,³⁸ another is the Ph.D. thesis of Spina.⁴⁶ Such experiments are often limited by either relatively low Mach numbers or difficulties taking measurements. Additionally, numerical simulations are hindered by insufficient separation of scales. While the present study is conducted at a relatively high Reynolds number for DNS, especially for compressible flow, there may still be insufficient separation of scales to allow a definitive investigation of power law behavior. While a fairly convincing, if somewhat noisy, k^{-1} region may be seen in Figure 10, there is no evidence of a $k^{-5/3}$ inertial subrange, as expected. This is due to insufficient separation of scales the present study higher Reynolds number as the present study, but at a considerably higher Reynolds number.

Despite the moderate Reynolds number of the current simulation, spectral analysis can still be of use, especially for gaining physical insight to be used in developing turbulence models and scaling arguments, and for assessing present modeling approaches. Below, in Figure 11, we can see the significant difference that the appropriate density scaling makes in the turbulence spectrum. In the left figure, (a), a spectrogram of the streamwise velocity is given in premultiplied form, and in the right figure, (b), the premultiplied spectrum is scaled by the density at that wall normal location. Note again that the scale is linear and arbitrary; plotting in this fashion highlights the relative energy at each length scale and wall normal location. The density scaling has the effect of emphasizing the energy in the logarithmic region, at larger wavelengths, consistent with findings in incompressible boundary layers.⁷

The isotropy of the small scales can be assessed using spectral analysis. Near the wall isotropy is not expected, since the no penetration condition constrains the wall normal fluctuations. In contrast, in the logarithmic region, if we have a sufficient separation of scales, then the small scales will be isotropic. In Figure 12 we have plotted an analogue to the anisotropy ratio, w_{rms}/u_{rms} and v_{rms}/u_{rms} , in a scale decomposed fashion, the anisotropy spectrum, ϕ_{ww}/ϕ_{uu} and ϕ_{vv}/ϕ_{uu} respectively. Ratios of 1, green on the color map, represent areas of near isotropy, while values near zero, blue, represent regions dominated by streamwise fluctuations and values much greater than one, red, represent areas dominated by wall-normal or spanwise fluctuations. In the logarithmic region between $z^+ = 30$ and $z^+ = 90$ we might expect that the smallest scales are isotropic. As we can see in Figure 12 there is indeed a tendency towards isotropy for length scales on the order of ten viscous length scales or less, although the very smallest scales seem to have an imbalance between the streamwise and wall normal components. Again this could be due to insufficient separation of scales or the grid convergence issues noted above in § III. Additionally, in Figure 12(a), we can see the influence of the wall, or the "splat" effect, clearly; at almost all scales energy in wall-normal fluctuations is dominated by the energy in the streamwise fluctuations very near the wall.

Next, we examine the contributions to the dominant Reynolds stress, $-\overline{\rho u'w'}$ at different scales and at different locations within the boundary layer. Again we may assume a Morkovin type scaling and say $-\overline{\rho u'w'} \approx -\overline{\rho u'w'}$. We examine contributions to this quantity using the coherence of streamwise and wallnormal velocity fluctuations and their phase. The coherence of two quantities, a and b, $\operatorname{coh}(a,b) = \gamma_{ab}^2$, is given in equation 2 where ϕ_{ab} is the cross power spectral density.

$$\gamma_{ab}^2 = \frac{|\phi_{ab}|^2}{|\phi_{aa}||\phi_{bb}|}\tag{2}$$

The phase is computed as $\theta_{ab} = \arg(\phi_{ab})$.

As can be seen in Figure 13(a) the u', w' coherence is moderate to very low at small length scales, but the larger length scales, those potentially associated with the near wall streaks at $z^+ = 12$ and those associated with the larger eddies of size $O(\delta)$ in the log and outer region of the flow contribute the most to the dominant stress. There is, however, one notable exception to this in the region very close to the wall, which achieves the highest level of coherence. This might be due again to the presence of the wall, or the "splat" effect. This near wall region should be quite quiescent and have little energy, and behave nearly as if it were incompressible since the local Mach number is very low. Thus, on the occasion that a wall-normal fluctuation develops in this near wall region, by the incompressible continuity equation for the instantaneous fluctuating quantities, equation 3, these wall-normal velocity fluctuations are transferred to the streamwise and spanwise components very near the wall.

$$\frac{\partial u'}{\partial x} + \frac{\partial v'}{\partial y} + \frac{\partial w'}{\partial z} = 0 \tag{3}$$

The phase plot, Figure 13(b), tells a similar story: It is primarily the scales of $O(\delta)$ that contribute to the broadband negativity of $\overline{u'w'}$. The white, black, and dark blue regions of Figure 13(b) indicate a phase of π between u' and w', meaning that when u' is positive w' is negative (a sweep event) and when u' is negative w' is positive (an ejection event). Thus it is predominantly the larger, coherent scales that contribute to the dominant Reynolds stress. Regions where θ_{uw} is between $-\pi/2$ and $\pi/2$ only correspond to small length scales, where there is little to no coherence indicating that these are the incoherent, noisy motions.

Next we turn our attention to the Strong Reynolds Analogy phase relationship. Morkovin postulated that the correlation coefficient of streamwise velocity fluctuations and temperature fluctuations should be nearly perfectly anti-correlated for a compressible boundary layer over a perfectly thermally insulating, smooth flat plate.²⁴ In Figure 14(a), we plot the coherence of these fluctuations, γ_{uT}^2 . Again, like the Reynolds stresses, the largest values of coherence happen for length scales of $O(\delta)$ and greater, and the level of coherence is nearly one in this region. Therefore these fluctuations are highly synchronized at large wavelengths as expected from theory and previous studies. Again, as in Figure 13(a), there is some evidence of a wall constrained "splat" region of high coherence very near the wall, and at small length scales. This region particularly stands out when we look at the phase, θ_{uT} , which is almost identically π every where in the flow except this region and in the noisy, small length scales in the outer portion of the boundary layer. In this near wall region the θ_{uT} is nearly zero, meaning that positive streamwise velocity fluctuations correspond to *positive* temperature fluctuations. From Figure 13(b) we know that we are still in a "sweep-ejection" regime and that streamwise fluctuations and wall normal fluctuations should have opposite signs. This implies that $\theta_{uT} \approx 1$ can be attributed to either or both of two factors: The mean temperature gradient has changed signs in this region, or these velocity fluctuations result in significant fluctuations in viscous heating in this region. Since we are using an isothermal wall condition which is slightly colder than the adiabatic wall temperature we assume that this effect is due primarily to the change in sign of the mean temperature gradient very near the wall.

V. Large Coherent Structures

The recent work by Hutchins and Marusic;⁷ Mathis, Hutchins and Marusic;¹⁸ and Marusic, Mathis and Hutchins⁸ has demonstrated the role that qualitative information about the coherent turbulent motions can lead to quantitative modeling approaches. As is evident in Figure 11 the present simulation is conducted at a Reynolds number too low to observe second peak occurring at the middle of the log layer in the spectrogram, thought to be the signature of "super structures." Never the less we may still observe the interaction of hairpin packets, to assess the validity of the VLSM model of Adrian and coworkers,⁴ and investigate the impact of the coherent structures on the wall, or their "foot print."

One approach to investigate the relationship between hairpin packets and "super structures" is the geometric packet finding algorithm of Ringuette, Wu and Martin,³¹ which is briefly summarized here. The algorithm scans (x, z) planes for ideal packets conforming geometrically to the model of Adrian, Meinhart. & Tomkins.⁴ Specifically, it searches for hairpin head, or transverse, vortices that are spaced close to one another ($\leq 0.5\delta$) in the streamwise direction and are arranged in a ramp-like formation with a small ($\leq 45^{\circ}$) downstream angle relative to the wall. The head vortices are identified using a threshold of the spanwise vorticity, ω_y , and the swirling strength, λ_{ci} ,⁴⁷ such that λ_{ci} must be greater than or equal to $4.5\overline{\lambda_{ci}}$, where the over-bar indicates the mean, and ω_y is greater than or equal to 2 standard deviations from the mean; only the region between the buffer layer $(z^+ = 30)$ and the boundary layer edge is considered for both computing the threshold quantities and finding hairpin packets. The algorithm has provisions for handling relatively large (wall-normal height > 0.1 δ , streamwise distance > 0.1 δ) structures that are occasionally identified by the thresholds, such as hairpin legs. The scheme checks whether or not a head vortex is above the leg, and does not consider the structure if no head is found. A further refinement of the algorithm, implemented in the current version, performs a least-squares-fit through the identified points of these large structures, and rejects the structure outright if the angle of the line is less than 25° . This removes the tendency of the algorithm to occasionally accept large, relatively horizontal shear layers with high ω_y or λ_{ci} as hairpin heads. The average packet properties determined using the algorithm before and after this improvement, however, show little change; the refinement is useful primarily for instantaneous visualizations. Figure 15, from Ringuette, Wu, and Martin,³¹ shows the results of this algorithm. Sub-figure (a) shows a three dimensional hairpin vortex packet found in a simulation at nearly matching conditions, and (b) shows a slice from the translucent plane highlighted in (a) with contours of spanwise vorticity. Boxes enclose the hairpin vortex heads found automatically by this algorithm. Finally, sub-figure (c) shows ticks, at two spanwise locations, indicating where hairpin packets were found. It is evident from this figure that hairpin packets do indeed align in the streamwise direction, giving rise to the "super structures."

Another approach to qualitatively and quantitatively observe hairpin packets, and their affect on the wall is the correlation method of Brown and Thomas. Brown & Thomas⁴⁸ identified the average large-scale coherent structure in the boundary layer associated with the unsteady signature of the wall shear stress, τ_w , by correlating τ_w at a single reference location with u at different wall-normal distances. Correlation profiles were constructed by varying the streamwise separation of u and τ_w , Δx , both by converting time into distance using Taylor's hypothesis of 'frozen convection' and physically shifting the streamwise measurement locations of u. They observed that the correlations peak at an increasing downstream distance with increasing wallnormal location, indicating a downstream-leaning coherent structure. Brown & Thomas⁴⁸ proposed that, if such a structure existed, then conditionally averaging the correlations on data traces that contained the structure would produce correlations of the same shape but with higher magnitudes, which they called 'enhanced' correlations. Their criterion used for the conditional averaging was that the correlation at $z/\delta =$ 0.25 be greater than twice the peak value at the Δx location of the peak, indicating a 'strong' event such as a turbulent burst in the lower part of the boundary layer. Using this conditional method, they found stronger correlations of the same shape, which provided evidence of a ramp-like coherent structures in the boundary layer. Since we have access to the entire flow field we can plot contours of this correlation coefficient, $R_{u,\tau_{uv}}$ in streamwise, wall-normal planes, and streamwise, spanwise planes, rather than 1 dimensional hot-wire traces. These correlation contours, like traditional two point velocity contours, give rise to evidence of three dimensional, down-stream leaning structures as can be seen in Figure 16. Further, this implies that the wall shear stress is dominated by such coherent structures. Iso-contours of this correlation coefficient can be thought of as regions of low momentum fluid encapsulated by hairpin like vortices. Counter rotating vortices near the wall, or hairpin legs, give rise to ejection events along the centerline of the streamwise, spanwise $R_{\mu,\tau_{w}}$ plane, causing a reduction in shear stress and transport of low momentum fluid away from the wall, causing positive R_{μ,τ_w} . On the outside of the counter rotating hairpin legs sweep events cause an increase in streamwise velocity while the shear stress, still being measured in the middle of the packet has a positive fluctuation, giving rise to negative R_{u,τ_w} . Additionally the spanwise aligned hairpin head also contributes to negative momentum fluctuations along the centerline.

These descriptions of hairpin packets, from the geometric analysis, the Brown and Thomas correlations, and the VLSM model of Kim and Adrian inspired an additional concept for investigating the relationship between the long low momentum streaks in the log layer, and the hairpin packets. We designed a two dimensional finite impulse response (FIR) filter using a anisotropic, bivariate Gaussian distribution with zero cross correlation to remove fluctuations which are smaller than hairpin packets. The filter removes 99.9% of energy at streamwise wavenumbers higher than $k_x \approx 2\pi/(5\delta_0)$ and 99.9% of energy at spanwise wavenumbers of $k_y \approx 2\pi/(0.65\delta_0)$. This filter is visualized to scale, next to the Brown and Thomas correlations in Figure 16(c).

We then applied this filter to two regions, streamwise momentum fluctuations in the middle of the logarithmic layer, and shear stress fluctuations at the wall. We hypothesized that if we apply this filter to momentum fluctuations in the log region we will select the meandering superstructures if they are indeed composed of hairpin packets. As can be seen in Figure 17(a,b) we can see that this is the case and we may even identify individual strong hairpin packets where this filtered field reaches local maxima. Figure 17(c,d) shows the wall shear stress with the same filter applied to it. Again we see long meandering regions with an almost perfect one to one correspondence with the log-layer "super structures." Figure 17 is evidence that superstructures do indeed have an important effect on the near wall cycle and drag production.

VI. Conclusions

We have shown, through spectral analysis and quantitative and qualitative techniques based on instantaneous time realizations, the strong dynamic influence of the largest structures. The large scales show strong contributions to the Reynolds shear stress, as well as the strong Reynolds analogy phase relationship between temperature and streamwise velocity fluctuations. Furthermore, at this Reynolds number and Mach number we present evidence that the "super structures" are due to streamwise aligned hairpin vortex packets, and that they contribute significantly to the near wall cycle, the wall shear stress, and thus the turbulence dynamics.

While the demands to accurately simulate the largest flow structures at these Reynolds numbers are stringent they are attainable. The present simulation is not entirely grid converged, but qualitative and some quantitative conclusions may still be drawn. Additionally, when undertaking an investigation of the large scale structures special attention should be paid to the inflow boundary condition if these are to accurately simulated.

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References

¹Xu, S. and Martín, M., "Assessment of Inflow Boundary Conditions for Compressible Turbulent Boundary Layers," *Physics of Fluids*, Vol. 16, No. 7, 2004, pp. 2623–2639.

²Theodorsen, T., "Mechanism of Turbulence," Proc. 2nd. Midwestern Conf. on Fluid Mech., Ohio State University, Columbus, Ohio, USA, 1952, pp. 1–19.

³Head, M. and Bandyopadhyay, P., "New aspects of turbulent boundary-layer structure," J. Fluid Mech., Vol. 107, 1981, pp. 297–338.

⁴Adrian, R., Meinhart, C., and Tomkins, C., "Vortex organization in the outer region of the turbulent boundary layer," J. Fluid Mech., Vol. 422, 2000, pp. 1–54.

⁵Jiménez, J., "The largest scales of turbulent wall flows," *Center for Turbulence Research, Annual Research Briefs*, Stanford University, 1998, pp. 137–154.

⁶Kim, K. C. and Adrian, R. J., "Very large-scale motion in the outer layer," Phys. Fluids, Vol. 11, No. 2, 1999, pp. 417–422.

⁷Hutchins, N. and Marusic, I., "Evidence of very long meandering features in the logarithmic region of turbulent boundary layers," *J. of Fluid Mechanics*, Vol. 579, 2007, pp. 1–28.

⁸Marusic, I., Mathis, R., and Hutchins, N., "Predictive model for wall-bounded turbulent flow," *Science*, Vol. 329, No. 5988, 2010, pp. 193–196.

⁹Tomkins, C. and Adrian, R., "Spanwise structure and scale growth in turbulent boundary layers," J. Fluid Mech., Vol. 490, 2003, pp. 37–74.

¹⁰del Álamo, J. C. and Jiménez, J., "Spectra of the very large anisotropic scales in turbulent channels," *Phys. Fluids*, Vol. 15, No. 6, 2003, pp. L41–L44.

¹¹Ganapathisubramani, B., Longmire, E. K., and Marusic, I., "Characteristics of vortex packets in turbulent boundary layers," J. Fluid Mech., Vol. 478, 2003, pp. 35–46.

¹²del Álamo, J. C., Jiménez, J., Zandonade, P., and Moser, R. D., "Scaling of the energy spectra of turbulent channels," J. Fluid Mech., Vol. 500, 2004, pp. 135–144.

¹³del Álamo, J. C., Jiménez, J., Zandonade, P., and Moser, R. D., "Self-similar vortex clusters in the turbulent logarithmic region," *J. Fluid Mech.*, Vol. 561, 2006, pp. 329–358.

¹⁴Guala, M., Hommema, S. E., and Adrian, R. J., "Large-scale and very-large-scale motions in turbulent pipe flow," J. Fluid Mech., Vol. 554, 2006, pp. 521–542.

¹⁵Hambleton, W. T., Hutchins, N., and Marusic, I., "Simultaneous orthogonal-plane particle image velocimetry measurements in a turbulent boundary layer," *J. Fluid Mech.*, Vol. 560, 2006, pp. 53–64.

¹⁶Flores, O., Jiménez, J., and del Álamo, J. C., "Vorticity organization in the outer layer of turbulent channels with disturbed walls," *J. Fluid Mech.*, Vol. 591, 2007, pp. 145–154.

¹⁷Balakumar, B. J. and Adrian, R. J., "Large- and very-large-scale motions in channel and boundary-layer flows," *Phil. Trans. R. Soc. A*, Vol. 365, 2007, pp. 665–681.

¹⁸Mathis, R., Hutchins, N., and Marusic, I., "Large-scale amplitude modulation of the small-scale structures in turbulent boundary layers," J. Fluid Mech., Vol. 628, 2009, pp. 311–337.

¹⁹Fernholz, H. H. and Finley, P. J., "A critical commentary on mean flow data for two-dimensional compressible turbulent boundary layers," *AGARDograph 253*, 1980.

²⁰Fernholz, H. H. and Finley, P. J., "A further compilation of compressible turbulent boundary layer data with a surveyof turbulence data," *AGARDograph 263*, 1981.

²¹Spina, E., Smits, A., and Robinson, S., "The physics of supersonic turbulent boundary layers," J. Fluid Mech., Vol. 26, 1994, pp. 287–319.

²²Smits, A. and Wood, D., "The response of turbulent boundary layers to sudden perturbations," Annual Review of Fluid Mechanics, Vol. 17, 1985, pp. 321–358.

²³Fernholz, H. H., Smits, A., Dussauge, J., and Finley, P., "A survey of measurements and measuring techniques in rapidly distorted compressible turbulent boundary layers," *AGARDograph* 315, 1989.

²⁴Smits, A. J. and Dussauge, J.-P., Turbulent shear layers in supersonic flow, Springer, 2nd ed., 2006.

²⁵Smits, A., Spina, E., Alving, A., Smith, R., Fernando, E., and Donovan, J., "A comparison of the turbulence structure of subsonic and supersonic boundary layers," *Phys. Fluids A*, Vol. 1, 1989, pp. 1865–1875.

²⁶Spina, E., Donovan, J., and Smits, A., "On the structure of high-Reynolds number supersonic turbulent boundary layers," J. Fluid Mech., Vol. 222, 1991, pp. 293–327.

²⁷Guarini, S., Moser, R., Shariff, K., and Wray, A., "Direct numerical simulation of supersonic turbulent boundary layer at Mach 2.5," *J. of Fluid Mech.*, Vol. 414, 2000, pp. 1–33.

²⁸Martín, M. P., "DNS of hypersonic turbulent boundary layers," AIAA Paper 2004-2337, 2004.

²⁹Martín, M. P., "Direct numerical simulation of hypersonic turbulent boundary layers. Part 1: Initialization and comparison with experiments," *J. Fluid Mech.*, Vol. 570, 2007, pp. 347–364.

 30 Pirozzoli, S., Grasso, F., and Gatski, T., "Direct numerical simulation and analysis of a spatially evolving supersonic turbulent boundary layer at M=2.25," *Phys. Fluids*, Vol. 16, No. 3, 2004, pp. 530 – 545.

³¹Ringuette, M. J., Wu, M., and Martín, M. P., "Coherent structures in direct numerical simulation of turbulent boundary layers at Mach 3," J. Fluid Mech., Vol. 594, 2008, pp. 59–69.

³²Adams, N., "Direct simulation of the turbulent boundary layer along a compression ramp at M = 3 and $Re_{\theta} = 1685$," J. of Fluid Mech., Vol. 420, 2000, pp. 47–83.

 33 Pirozzoli, S. and Grasso, F., "Direct numerical simulation of impinging shock wave/turbulent boundary layer interaction at M=2.25," *Phys. Fluids*, Vol. 18, No. 6, 2006.

³⁴Wu, M. and Martín, M., "Direct numerical simulation of shockwave and turbulent boundary layer interactions over a compression ramp," *AIAA J.*, Vol. 45, 2007, pp. 879–889.

³⁵Wu, M. and Martín, M., "Analysis of shock motion in shockwave and turbulent boundary layer interaction using direct numerical simulation data," *J. of Fluid Mechanics*, Vol. 594, 2008, pp. 71–83.

³⁶Taylor, E. M., Grube, N., and Martín, M. P., "Evaluation of traditional and shock-confining LES filters using data of compressible turbulence," *AIAA Paper Number 2007-4197*, 2007.

³⁷Grube, N., Taylor, E. M., and Martín, M. P., "Numerical Investigation of Shock/Isotropic Turbulence Interactions," 49th AIAA Aerospace Sciences Meeting including the New Horizons Forum and Aerospace Exposition. Orlando, Florida. Jan 4 - 7.

³⁸Elsinga, G., Adrian, R., Van Oudheusden, B., and Scarano, F., "Three-dimensional vortex organization in a high-Reynolds-number supersonic turbulent boundary layer," *J. of Fluid Mech.*, Vol. 644, 2010, pp. 35–60.

³⁹Schrijer, F., Scarano, F., and van Oudheusden, B., "Application of PIV in a Mach 7 double-ramp flow," *Experiments in Fluids*, Vol. 41, 2006, pp. 353–363.

⁴⁰van Oudheusden, B., "Principles and application of velocimetry-based planar pressure imaging in compressible flows with shocks," *Experiments in Fluids (electronic Open Access)*, 2008.

⁴¹Humble, R., Scarano, F., and van Oudheusden, B., "Experimental study of an incident shock wave/turbulent boundary layer interaction using PIV," *AIAA Paper Number*, Vol. number 2006-3361, 2006.

⁴²Ganapathisubramani, B., Clemens, N., and Dolling, D., "Large-scale motions in a supersonic turbulent boundary layer," J. Fluid Mech., Vol. 556, 2006, pp. 1–11.

⁴³Ganapathisubramani, B., Clemens, N., and Dolling, D., "Effects of upstream boundary layer on the unsteadiness of shock-induced separation," J. Fluid Mech., Vol. 585, 2007, pp. 369–394.

⁴⁴Martin, M. P., Taylor, E. M., Wu, M., and Weirs, V. G., "A Bandwidth-Optimized WENO Scheme for the Direct Numerical Simulation of Compressible Turbulence," J. of Comp. Physics, Vol. 220, 2006, pp. 270–289.

⁴⁵Simens, M., Jimnez, J., Hoyas, S., and Mizuno, Y., "A high-resolution code for turbulent boundary layers," J. of Comp. Physics, Vol. 228, No. 11, 2009, pp. 4218–4231.

⁴⁶Spina, E., Organized structures in a supersonic turbulent boundary layer., Ph.D. thesis, Princeton University, Princeton, New Jersey, 1988.

⁴⁷Zhou, J., Adrian, R., Balachandar, S., and Kendall, T., "Mechanisms for generating coherent packets of hairpin vortices in channel flow," *J. Fluid Mech.*, Vol. 387, 1999, pp. 353–396.

⁴⁸Brown, G. L. and Thomas, A. S. W., "Large structure in a turbulent boundary layer," *Phys. Fluids*, Vol. 10, No. 2, 1977, pp. 243–251.



(a) Theodorsen's hairpin vortex.² The arrows on either side of the hairpin indicate the direction of the flow.

(b) Very large scale motion model of Adrian *et al.*⁴ in which hairpin packets align to produce the long, low-momentum streaks in the logarithmic layer.





Figure 2. Conceptual drawing of "super-structures" in a turbulent boundary layer from Marusic et al.⁸



Figure 3. Computational domain for the DNS, truncated in the z direction. Flow is from left to right. The rescaling and inflow planes have been highlighted.



Figure 4. C_f vs Re_{θ} and $\pm 5\%$ of the Van Driest II semi-empirical predictions. Each symbol represents an instant in time.



Figure 5. Van Driest-transformed velocity profile at various streamwise locations.



Figure 6. Two-point auto-correlations as a function of spanwise separation taken at various streamwise locations in the outer region of the boundary layer.



Figure 7. Premultiplied pressure and streamwise velocity spectrum at various wall normal locations. Wavenumber spectrum obtained using Taylor's hypothesis. Arbitrary, linear scale.



Figure 8. Premultiplied pressure spectrogram, linear, arbitrary-scale color map.



Figure 9. Two numerical schlieren (equation 1) images taken in the same streamwise, wall-normal plane at two instances in time, separated by about $\Delta t_{a,b} = t_b - t_a \approx \lambda_0 / U_{\infty}$, the rescaling convective time scale.



Figure 10. Log-log plot of the streamwise velocity power spectral density taken from the middle of the logarithmic layer. The k^{-1} region appears convincing, but the inertial subrange appears to small to see evidence of a $k^{-5/3}$ region. The lack of a $k^{-5/3}$ region may be due to the lack of grid convergence which is providing unphysical dissipation, at scales larger than the true dissipative turbulent length scales.



Figure 11. Premultiplied streamwise velocity wave-number spectrogram. Arbitrary, linear color map used for both, but the right hand plot is scaled the local density.



Figure 12. Anisotropy spectrum. Left: w, right: v.



Figure 13. The dominant Reynolds stress, $\overline{u'w'}$. Left: coherence. Right: phase.



Figure 14. Strong Reynolds Analogy phase relation, $\overline{u'T'}$. Left: Coherence, Right: phase.



(a) Hairpin Packet conforming to the model of Adrian⁴ found in a Mach 2.9 boundary layer using the automated algorithm of Ringuette, Wu & Martin. See Figure 1(b)





(c) Tickmarks indicating the presence of hairpin vortex packets found by searching two streamwise, wall-normal planes (indicated by the black lines) overlaying contours of streamwise velocity deficit in the lagarithmic layer.

Figure 15. Figures reproduced from Ringuette, Wu & Martin³¹ conducted at the same Mach number, and slightly lower Reynolds number.



(a)



(b)



Figure 16. Contours of Brown & Thomas correlations, R_{u',τ'_w} in the (a) streamwise, wall-normal plane and (b) streamwise, spanwise plane. (c) plots a two dimensional finite impulse response filter generated with a bivariate Gaussian distribution to filter out noisy fluctuations which have geometries smaller than a hairpin packet.



Figure 17. (a) unfiltered and (b) filtered streamwise momentum fluctuations in the middle of the logarithmic layer. (c) *filtered* and (d) *unfiltered* wall shear stress fluctuations. All filtering was done with the filter shown in Figure 16(c).